University of Saskatchewan
Department of Mathematics & Statistics
Mathematics 110.3

Time: 3 hours | Final Examination | 9am, December 9, 2000

CLOSED BOOK EXAMINATION - NO CALCULATORS ALLOWED

Name: _____ Student #: _____ Math 110 section # _____

PART I

Questions in this part will be marked right or wrong. Please carefully write your answers in the spaces provided.

[5] 1. (a) $\lim_{t\to -3}(t^2-2t) =$ _____

(b) $\lim_{x\to 2}\dfrac{x-2}{x^3-2x^2+x-2} =$ _____

(c) $\lim_{x\to 0}\dfrac{x}{\sqrt{x+1}-1} =$ _____

(d) $\lim_{h\to 2}\dfrac{\frac{1}{h}-\frac{1}{2}}{h-2} =$ _____

(e) $\lim_{x\to 1^-}\dfrac{|x-1|}{x^2-1} =$ _____

[3] 2. (a) For what x-value(s) does the graph of $y=\dfrac{x+1}{x^2-2x-3}$ have a vertical asymptote?

_____

(b) Find $\lim_{x\to\infty}\dfrac{4x-\sqrt{x^2+x}}{8x+5}$. _____

(c) Find $\lim_{t\to 0^-} e^{1/t}$. _____

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[18] 3. Carry out the indicated differentiations. It is not necessary to simplify your answers.

(a) If $y = (2x^2 - x)^3$ then $\frac{dy}{dx} =$ _____

(b) If $f(t) = \sec 3t$ then $f'(t) =$ _____

(c) If $u = \frac{t^2 + 2t}{t^3 - t + 1}$ then $\frac{du}{dt} =$ _____

(d) If $g(x) = e^x \sin x$ then $g'(x) =$ _____

(e) If $h(x) = x^2 + 2^x + 2^2$ then $h'(x) =$ _____

(f) If $y = x^{2x+1}$ then $y' =$ _____

(g) If $z = \sin^{-1}(\sqrt{x})$ then $\frac{dz}{dx} =$ _____

(h) If $p(x) = \cos(\sqrt[3]{x + \tan x})$ then $p'(x) =$ _____

(i) If $y = \ln(\frac{u}{2u+1})$ then $\frac{dy}{du} =$ _____

[10] 4. (a) What is the domain of the function $\ln(4 - x^2)$? _____

(b) Complete the following statement:
If f has a local extremum at the number c then either $f'(c) = 0$ or _____

(c) Find an antiderivative $F(x)$ of $f(x) = x^2 + \sin x$ that satisfies $F(\pi) = 1$.

_____

(d) Use the Fundamental Theorem of Calculus to evaluate $\int_1^3 (x - \frac{1}{x})dx$. _____

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(e) Complete the following statement:
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) then there exists a number c in the open interval (a, b) such that $f'(c) =$ _____

PART II

Please provide carefully written answers to questions 5 through 12 in an answer booklet.

[6] 5. Use the formal definition of the derivative (that is, work from first principles) to find the slope of the tangent line to the graph of $y = x^2 - x$ at the point $(3, 6)$. (No marks will be given for using the rules of differentiation.)

[6] 6. A water tank has the shape of an inverted circular cone with base radius 2 m and height 5 m. If water is being pumped out of the tank at the rate of 1 m^3 per min, find the rate at which the water level is falling when the water is 2 m deep. (Hint: The volume of a circular cone with height h and base radius r is $V = \frac{1}{3}\pi r^2 h$.)

[6] 7. Consider $f(x) = x^4 - 2x^2 + 3$.
(a) Identify the intervals where $f(x)$ is increasing or decreasing.
(b) Identify any local maxima or minima of $f(x)$.
(c) Identify the intervals where $f(x)$ is concave up or down.
(d) Identify all inflection points of $f(x)$.
(e) Sketch the graph of $y = x^4 - 2x^2 + 3$.

[6] 8. A farmer wants to enclose a rectangular area of 288 m^2 with fencing and then divide it into 3 pens with fences parallel to one side of the rectangle. What is the least amount of fencing he needs to use?

[6] 9. Find the equation of the tangent line to the graph of $x^2y + y^2 + x = 7$ at the point $(2, 1)$.

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[6] 10. Consider the function

$$f(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ x+1 & \text{if } 0 < x \leq 2 \\ 7 - x^2 & \text{if } x > 2 \end{cases}.$$

(a) Sketch the graph of f

(b) For what values of x is f continuous? For what values of x is f differentiable?

[6] 11. Let $f(x) = x^3 + 2x - 1$.

(a) Carefully explain how you know that the equation $f(x) = 0$ has a root in the interval $[0, 1]$.

(b) Estimate the root of the equation $f(x) = 0$ by starting with the initial guess $x_1 = 1$ and applying one step of Newton's method. Leave your answer in fractional form.

[6] 12. Let $f(x) = xe^{-x}$ for $x \in [-1, 2]$.

(a) Find all points $x \in [-1, 2]$ that are critical numbers for f.

(b) What is the absolute maximum value and the absolute minimum value of $f(x)$ for $x \in [-1, 2]$?

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PART III

Please provide a carefully written answer to ONE of the following TWO questions in an answer booklet.

EITHER

[6] 13. A ball is thrown upward with an initial velocity of 64 ft/sec from the top of a tower 80 ft above the ground.

(a) Find its height above the ground t seconds later.

(b) When does it reach its maximum height?

Hint: The acceleration due to gravity is 32 ft/sec^2.

OR

[6] 14. (a) Approximate the definite integral $\int_0^2 (x^2 + x)dx$ using a Riemann sum with left endpoints and 4 equal subdivisions.

(b) Draw a diagram to illustrate the approximation in part (a). Is the approximation too large or too small? Explain.

[90] Total

** The End **